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Aut _ FM18

میان ترم   4_5 نمره   ←   اواخر آبان _ اوایل آذر

پایان ترم   9_10 نمره

فعالیت کلاسی   1_2 نمره

TA + کوییز   3 نمره

Hw   1 نمره   ←   اوراق

1. Introduction to fluid mechanic, Fox   Edition 6

2. Fluid mechanic, Cengel

3. Unit opration in chemical Engineering, Mc cabe

4. Fluid Mechanic, Streeter

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Chapter 1 : Fluid Properties

1. Conservation of mass → $mass_{in} - mass_{out} + \text{mass of}_{\text{Generate}} = \text{mass of}_{\text{accumulation}}$

2. Conservation of energy → 1 st term of termodinamics

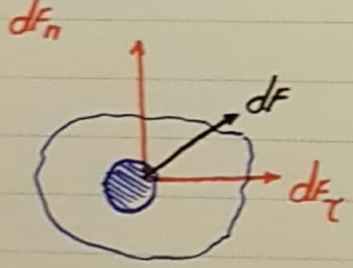
$$Q - W = \Delta U$$

3. 2^{nd} law of termodinamics → $\Delta S \geq 0$ → isolate system
 → $\Delta S \geq \frac{Q}{t}$ → closed system

4. Newton's law of motion → $F = ma$

Mechanics : The study of Force & motion

Fluid : A substance ~~with~~ which continously deforms when subjected to a Shear force
(Shear → برشی)



-(Presure) = normal stress

* ما به صورت قرارداد فشار را به سمت پایین در نظر میگیریم.

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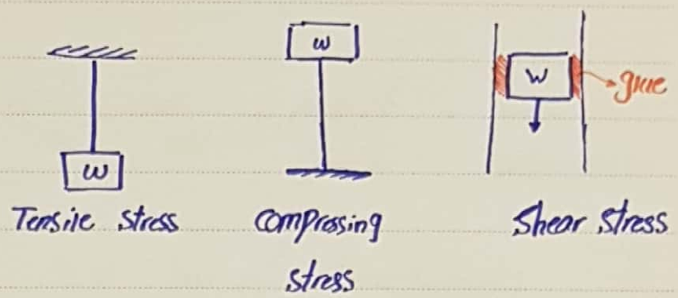
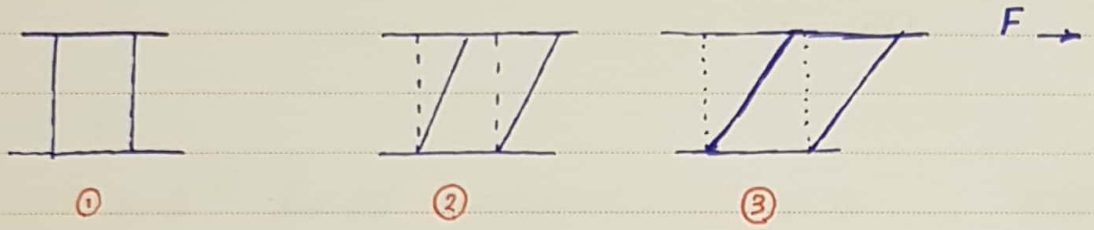
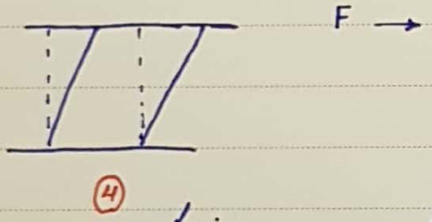
Stress
- normal stress
- tensile Stress
- Shear stress





در تمام شکل های بالا نیرو وارد می شود. $F_3 > F_2 > F_1$

با افزایش نیرو به صورت پیوسته شکل تغییر پیدا می کند پس می توان گفت ماده یک سیال است



با این که نیروی F_4 از تمام نیروهای قبلی بیشتر است ولی تغییر شکل نداده پس نمی توانیم بگوییم

که ماده ی ما سیال است. بلکه می توانیم بگوییم ماده یک جامد است.

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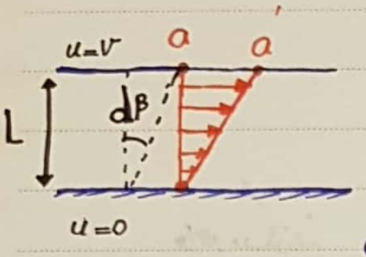
Viscosity : A measure of resistance to flow



1. laminar flow — آرام یا لایه لایه حرکت می کند

با حرکت از لایه ی بالایی به لایه های پایینی سرعت کاهش می یابد چون بین لایه های سیال ویسکوزیته وجود دارد.

$$u(y) = \frac{V}{L} y$$

$$d\beta \to 0 \Rightarrow d\beta = \tan\beta = \frac{da}{L} = \frac{V\,dt}{L}$$

$$\Rightarrow \frac{du}{dy} = \frac{V}{L}$$

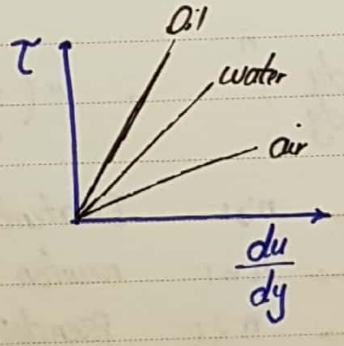
$$\Rightarrow \frac{d\beta}{dt} = \frac{V}{L} = \frac{du}{dy}$$

Newtonian fluid :

$$\tau = \frac{F}{A}$$

$$\tau \propto \frac{du}{dy}$$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$



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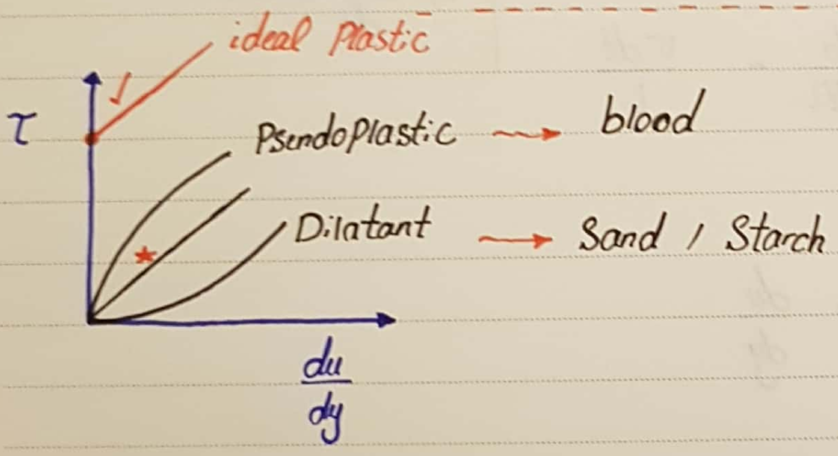
$\tau = \mu \frac{V}{L} \rightarrow \frac{N}{m^2}$ $\xrightarrow{\tau = \frac{F}{A}}$ $F = \mu \frac{V}{L} A \rightarrow Pa.S$ یا $\frac{kg}{m.S}$

واحد ویسکوزیته Poise یا Centi-Poise است.

$Poise \simeq 0.1\ Pa.S$

$Centi\text{-}Poise = 10^{-3}\ Pois$



$*: \ \tau = \mu \frac{du}{dy}$

$\checkmark: \ \tau = \tau_p + \mu \frac{du}{dy}$

$\tau = \mu \left(\frac{du}{dy}\right)^n = \mu \left(\frac{du}{dy}\right)^{n-1} \left(\frac{du}{dy}\right) = \eta \frac{du}{dy}$

n:
- $n > 1$ Dilatent
- $n = 1$ newton
- $n < 1$ Pseudoplastic

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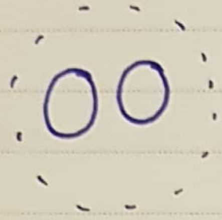
$\frac{\mu}{\rho} = \nu$ Kinematic Viscosity

$\rightarrow \frac{m^2}{s} \rightarrow$ Stoke $(1\frac{cm^2}{s})$

سیال پیوسته: سیستم که در نظر می‌گیریم به قدری بزرگ است که فاصله‌ی بین مولکول‌ها نتوانند یک گپ به وجود بیاورد

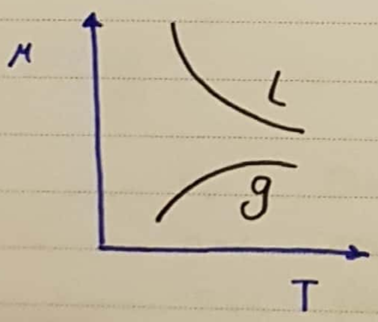


اگر یک دماسنج را بین این دو مولکول قرار دهیم اگر فاصله‌ی بین مولکول‌ها را
در نظر نگیریم یعنی به نوعی سیال را پیوسته در نظر بگیریم می‌توان از
تفاوت دما بین دو مولکول صرف نظر کرد.

$T \propto \mu$
- liquid → intermolecular ↑T IMF↓ → μ↓
- gasses → molecular collisions ↑T MC↑ → μ↑

فشارهای کم و متوسط روی مایعات، گازها تأثیر زیادی ندارد ولی فشار بالا ویسکوزیته‌ی گازها را بالا می‌برد

ولی روی ویسکوزیته‌ی مایعات تأثیری ندارد.



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Continuum: A System which is studied is large enough (This system is far longer than the space between molecules)

* برای گازها گفتیم:

T ↑ μ ↑

برای گازها رابطه‌ی زیر برقرار است:

$$\mu_{gas} = \frac{a T^{1/2}}{1 + \frac{b}{T}}$$

$a = 1.46 \times 10^{-6} \ \frac{kg}{(m.s.K^{1/2})}$

$b = 110.4 \ K$

* برای مایعات گفتیم:

T ↑ μ ↓

برای مایعات رابطه‌ی زیر برقرار است:

$$\mu_{lq} = a \times 10^{\frac{b}{T - C}}$$

$a = 2.41 \times 10^{-5} \ \frac{N.S}{m^2}$

$b = 247.8 \ K$

$C = 140 \ K$

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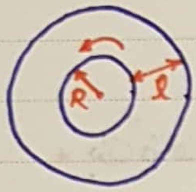
Viscometer : دستگاهی است برای اندازه‌گیری ویسکوزیته .

دستگاهی است که از دو استوانه‌ی تو در تو تشکیل شده است .



$$Torque = F \times R$$

$$= \tau \times A \times R = \mu \cdot \frac{v}{l} \times A \times R = \mu \frac{\omega \times R}{l} \times A \times R$$

$$= \mu \frac{2\pi n \times R \times 2\pi R \times L \times R}{l} = \frac{\mu 4\pi^2 n R^3 L}{l} = T$$

* ما سرعت را به صورت خطی فرض کردیم . این فرض زمانی درست است که $l \ll L$ باشد

Density :

$$\rho = \frac{m}{V}$$

$$\rho_{water, 20^\circ C} = 998 \frac{kg}{m^3}$$

$$\rho_{water, 75^\circ C} = 975 \frac{kg}{m^3}$$

* اثر تغییرات دما در مایع بیشتر از فشار در مایع است .

$\frac{1}{\rho}$ = specific volume = حجم ویژه

$S.G = \frac{\rho_x}{\rho_{H_2O}}$ $\gamma = \rho \times g$ specific wight : جرم ویژه

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Fluid
- Compressible → برای افزایش فشار باید از دستگاه Compressor استفاده کنیم → گازها
- in compressible → برای افزایش فشار باید از دستگاه Pomp استفاده کنیم → مایعات

$P = \rho R' T$ $\qquad R' = \frac{R}{M} = \frac{8314}{M}$

Coefficient of compressibility ، ضریب تراکم پذیری :

$$k = \left[- \frac{\partial P}{\left(\frac{\partial v}{v}\right)} \right]_T = -v \frac{\partial P}{\partial v} \simeq -v \frac{\Delta P}{\Delta v} = \rho \frac{\partial P}{\partial \rho}$$

→ Bulk modulus of elasticity

$$K_{water} = -\frac{210 atm}{-0.01} = 21000\ atm$$

$$K_{\text{ایده‌آل}} = P$$

Vapor Pressure :

مولکول‌ها در هر دمایی بخار می‌شوند. به فشاری که در حین تبدیل مایع به گاز تبادل می‌شود فشار بخار می‌گویند. فشار بخار به دما و فشار بستگی دارد.

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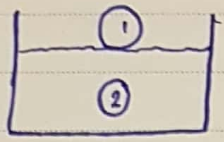
Surface tension and capillary effect:

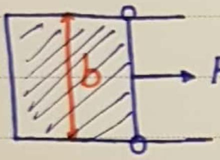


عدم توازن نیروهای بین مولکولی، عدم تقارن کلی در سطح باعث می‌شود نیرویی از پایین به بالا وارد می‌شود.

* streched elastic membrane

$$\begin{cases} \sigma_{water} = 0.0728\ N/m = J/m^2 \\ \sigma_{Hg} = 0.44\ N/m \end{cases}$$

* جیوه بیشتر از آب تمایل به کروی شدن دارد.



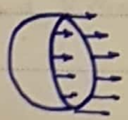
$$\sigma = \frac{F}{2b} = \frac{W}{A}$$

$T \uparrow \qquad \sigma \downarrow$

اندازه‌ی قطره با نیروی بین مولکولی تناسب دارد.

$P_{in} > P_{out}$ قطره بزرگ می‌شود

$P_{in} < P_{out}$ قطره کوچک می‌شود



$$F_1 = F_2$$

$$\sigma (2\pi R) = \Delta P (\pi R^2) \longrightarrow \frac{2\sigma}{R} = \Delta P$$

Bubble: $\frac{4\sigma}{R} = \Delta P$?

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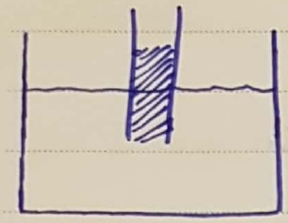
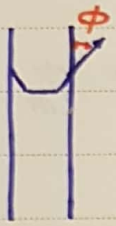


Cohesive → like moleculs

Adhesive → unlike moleculs

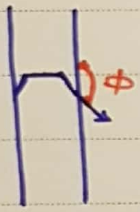
?? اگر قطر دهانه‌ی لوله خیلی بیشتر از 1 cm باشد Cohesive انجام نمی‌شود یعنی آب بالا نمی‌آید



Φ = Contact angle $< 90^\circ$

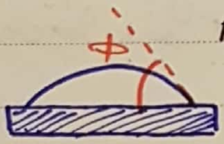
$$\sigma(2\pi R)\cos\Phi = \rho v g = \rho_r \pi R^2 h g \Rightarrow h = \frac{2\sigma\cos\Phi}{\rho R g}$$

زاویه‌ی سطح مایع: زاویه‌ای است که مایع با دیواره می‌سازد ← Φ



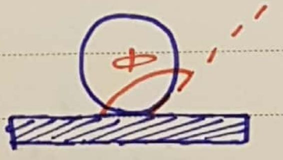
$\Phi > 90^\circ$

آب تا جایی بالا می‌رود که برآیند نیروهای Cohesive، وزن خنثی شود.



$\Phi < 90$

High Surface Energy



$\Phi > 90$

low Surface Energy

هر دو ماده آب هستند ولی به دلیل تفاوت در سطح یکی پخش می‌شود ولی دیگری کروی

* یکی از راه‌های کم کردن Surface tension استفاده کردن از Surfacetants است.

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Chapter 2 : Fluid statics

1. Fluid layers don't move

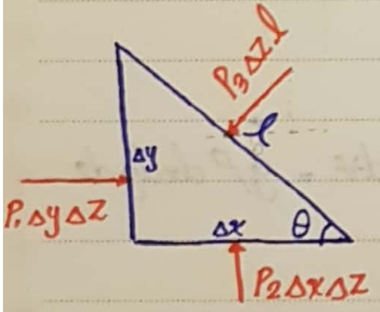
2. No Shear Stress or force

Pressure $= \dfrac{F}{A}$ Pa, bar, psi, tor, atm, ...

$$P_{abs} = P_G + P_{atm}$$

* P_G هم می تواند مثبت باشد، هم منفی

Pressure at a Point :



عمق : ΔZ

قانون دوم نیوتن : $\sum F = ma$ $\xrightarrow{static}$ $\sum F = 0$

$\Rightarrow \sum F_x = 0 \Rightarrow P_1 \Delta y \Delta Z - P_3 \Delta Z \, l \, \underline{\sin\theta} = 0$ ($l \sin\theta = \Delta y$)

$$\Rightarrow P_1 = P_3 \quad (1)$$

$\Rightarrow \sum F_y = 0 \Rightarrow P_2 \Delta x \Delta Z - P_3 \Delta Z \, \underline{l \cos\theta} - \rho g \left(\frac{1}{2} \Delta x \Delta y \Delta Z\right) = 0$ ($l \cos\theta = \Delta x$)

$\Rightarrow P_2 - P_3 - \frac{1}{2} \rho g \Delta y = 0$
$\Delta y \to 0$

$$\Rightarrow P_2 = P_3 \quad (2)$$

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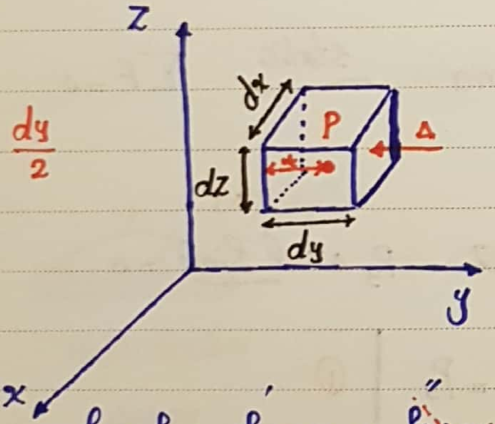
①, ② $\Longrightarrow$ $P_1 = P_2 = P_3 = P$

* The pressure at a point of fluid has the same magnitude in all directions.

Force
- Surface force
- body force → gravity force

$$F_t = F_s + F_b$$

The basic equation of fluid static:



$$\vec{dF_B} = \vec{g}\,dm = \vec{g}\rho\,d\forall = \vec{g}\rho\,dx\,dy\,dz$$

$$\vec{dF_{sy}} = \left(P - \frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right) dz\,dx$$

نیرویی که به ضلع سمت چپ وارد می شود، از بسط تیلور بدست می آید:

$$f(x) = f(a) + f'(x)(x-a) + \frac{f''(x)}{2}(x-a)^2$$

$$\rightarrow \vec{dF_s} = \vec{dF_{sx}} + \vec{dF_{sy}} + \vec{dF_{sz}} = -\left(\frac{\partial P}{\partial x}\hat{i} + \frac{\partial P}{\partial y}\hat{j} + \frac{\partial P}{\partial z}\hat{k}\right) dx\,dy\,dz$$

Δ: $\vec{dF_{sy}} = \left(P + \frac{\partial P}{\partial y}\cdot\frac{dy}{2}\right) dz\,dx$

$$\Rightarrow \; -\nabla P \, d\forall = d\vec{F}_s$$

$$d\vec{F}_t = d\vec{F}_s + d\vec{F}_b \quad \Rightarrow \quad d\vec{F}_t = (-\nabla P + \rho \vec{g}) \, d\forall$$

$$\xrightarrow{\text{static}} \; a = 0 \;\rightarrow\; d\vec{F}_t = 0 \;\rightarrow\; \nabla P = \rho \vec{g}$$

$$\Rightarrow \begin{cases} -\dfrac{\partial P}{\partial x} + \rho g_x = 0 \quad (g_x \to 0) \\[2mm] -\dfrac{\partial P}{\partial y} + \rho g_y = 0 \quad (g_y \to 0) \\[2mm] -\dfrac{\partial P}{\partial z} + \rho g_z = 0 \end{cases} \quad \Rightarrow \quad -\dfrac{dP}{dz} + \gamma = 0 \;\star$$

★ The pressure of continues fluid at rest is always the same for the same elevation.

شرایط ★ :

1) Static fluid

2) The Z-axis is upward

3) $\vec{g}$ is the only body force

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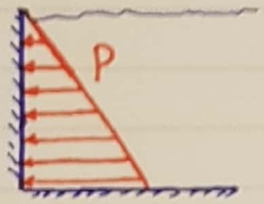
$$\frac{dp}{dz} = -\gamma = -\rho g$$



★ Profile سرعت به صورت خطی است :

$$P = -\gamma z = -\rho g z$$

★ مساحت شکل ارتباطی به فشار ندارد

Pressure variations in Compressible fluid :

fluid:
- $\rho = Cte$ for incompressible
- $\rho \neq Cte$ for compressible

$$dp = -\gamma dy \Rightarrow dy = -\frac{dp}{\gamma} = -\frac{dp}{\rho g}$$

$$\Rightarrow dy = \frac{-P_0}{\rho g} \cdot \frac{dp}{P} \xrightarrow{\text{انتگرال}} P = P_0 \exp\left(\frac{-(y-y_0)}{\frac{P_0}{\rho_0 g}}\right)$$

$T = T_0 - my$

$+0.0065 \frac{C}{m}$

$$dp = -\rho g dy = -\frac{P}{R(T_0 - my)} g\, dy$$

$$\xrightarrow{\text{انتگرال}} P = P_0 \left(1 - \frac{mq}{T_0}\right)^{\frac{g}{mR}} = P_0 \left(\frac{T}{T_0}\right)^{\frac{g}{mR}}$$

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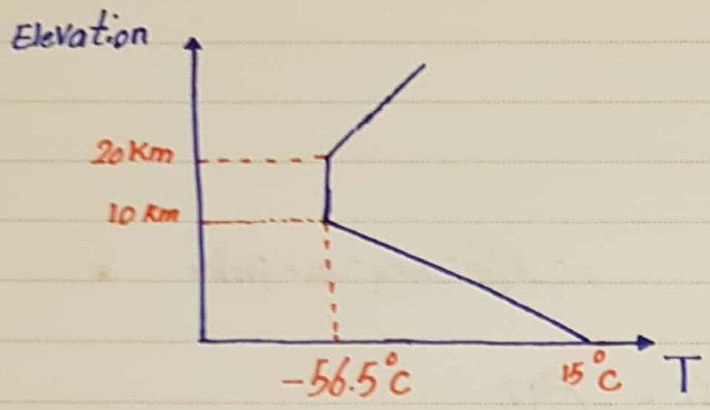
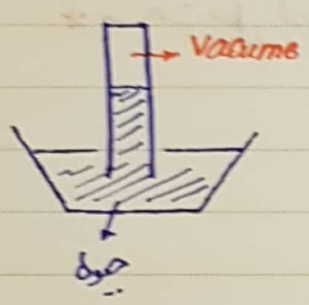


مانومتر : اندازه گیری اختلاف فشار .

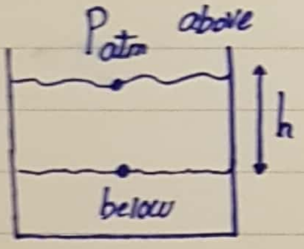


→

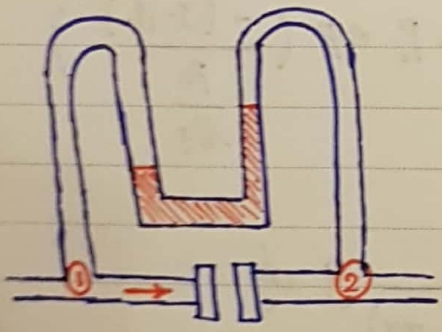
$W = \rho (A \times h)\, g$

$P_{atm} \times A$

$\Rightarrow P_{atm} = \rho g h$



$\rightarrow P_h = P_{atm} + \rho g h \Rightarrow P_{below} = P_{above} + \rho g h$



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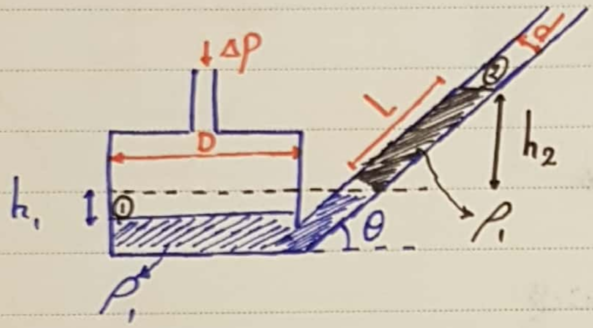
manometer
- High Pressure → $\rho \uparrow$ → $\rho_{Hg} = 13.6\, \rho_{H2O}$
- low Pressure → inclined manometer

Fox, Edit 6th, Ex 3.2:



دقت

$$S = \frac{L}{h} \quad ?$$

$$P_1 - \rho_1 g h_1 - \rho_1 g h_2 = P_2 \Rightarrow P_1 - P_2 = \rho_1 g (h_1 + h_2)$$

$$\frac{\pi D^2}{4} h_1 = \frac{\pi d^2}{4} L \rightarrow h_1 = L \left(\frac{d}{D}\right)^2$$

$$\rightarrow \Delta P = \rho_1 g \left(L \left(\frac{d}{D}\right)^2 + L \sin\theta \right) \Rightarrow L = \frac{\Delta P}{\rho_1 g \left(\sin\theta + \left(\frac{d}{D}\right)^2\right)}$$

$$S = \frac{L}{h} = \frac{L}{\frac{\Delta P}{\rho g}} = \frac{1}{S.G_1 \left[\sin\theta + \left(\frac{d}{D}\right)^2\right]} \checkmark$$

max S: $S.G = 0.8$, $\frac{d}{D} = 0.1$, $\theta = 10^\circ$ ⇒ $S = 6.81$

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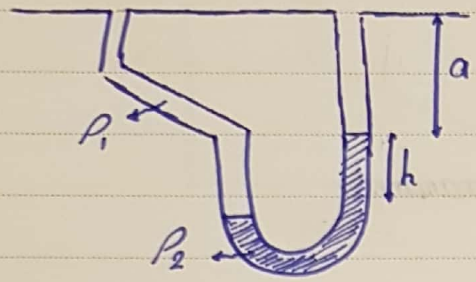
fluid →



$$P_1 + \rho_1 g(h+a) - \rho_2 g(h) - \rho_1 g a = P_2$$

$$\rightarrow P_1 + \rho_1 gh - \rho_2 gh = P_2$$

$$\rightarrow P_1 - P_2 = (\rho_2 - \rho_1)gh$$

* Bourdon gauge: → پر استفاده ترین
* Strain - gauge: → دقت بسیار بالا
* Pizo - electric: → برای فشارهای پایین استفاده می‌شود

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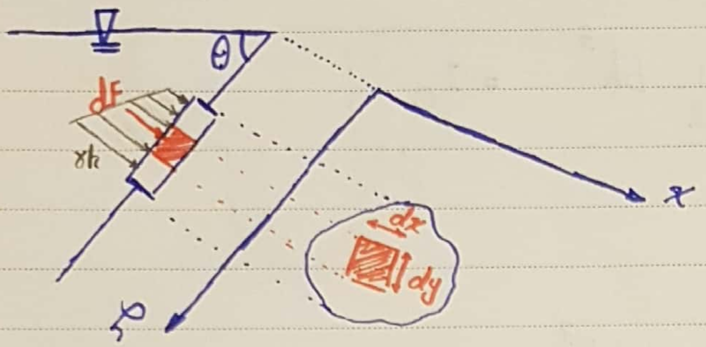
Hydrostatic force:



$$F = \int P\,dA = \int (P_0 + \rho g h)\,dA$$

$$= \int (P_0 + \rho g y \sin\theta)\,dA$$

$$= \int P_0\,dA + \rho g \sin\theta \int y\,dA$$

$$= P_0 A + \rho g \sin\theta \int_A y\,dA$$

first moment of area

$\int y\,dA = Y_C A$ (Centroid) $\Rightarrow F = P_0 A + \rho g \sin\theta \cdot y_C A = P_0 A + \rho g h_C A$

$$\Rightarrow F = (P_0 + \rho g h_C) A = P_C A$$

resultant force

* The resultant force on a plane surface completly submerged in a homugen fluid (ρ = constant) is equal to the product of pressure imposed on a centroid and surface area.

گشتاور : $Y_P \times F_R = \int y\,dF = \int y\,P\,dA = P_0 \int y\,dA + \rho g \sin\theta \int y^2\,dA$

$$Y_P \times F_R = P_0 y_C A + \rho g \sin\theta\, I_{xx}$$

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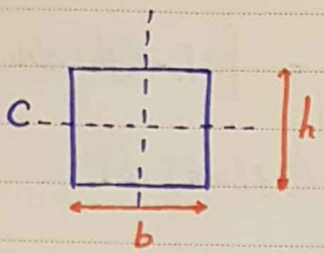
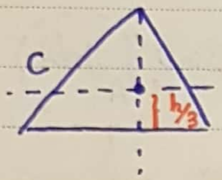
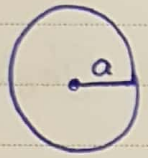
I_{xx} برای مستطیل :



$$I_{xx} = \frac{1}{12} bh^3 = I_C$$



$$I_{xx} = \frac{1}{36} bh^3 = I_C$$



$$I_{xx} = \pi \frac{a^4}{4} = I_C$$

$$\Rightarrow y_p = y_c + \frac{I_{xx}}{y_c A}$$

تئوری محورهای موازی : $I_{xx} = I_C + y_c^2 A$

$$y_p F_R = P_0 y_c A + \rho g \sin\theta \left(I_C + y_c^2 A \right) = y_c A \left(P_0 + \rho g \sin\theta \, y_c \right) + \rho g \sin\theta \, I_C$$

$$\rightarrow y_p = y_c + \frac{I_C \rho g \sin\theta}{F_R}$$

F_R (net force) $\rightarrow \rho g \sin\theta \, y_c A$

$$I_C > 0 \longrightarrow y_p > y_c$$

$$x_p = x_c + \frac{\rho g \sin\theta \, I_{xy}}{F_R}$$

$I_{xy} = 0 \leftarrow$ متقارن باشد

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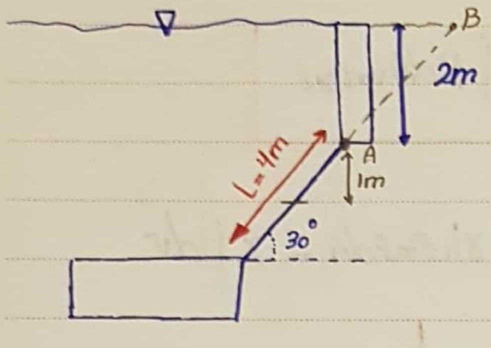
(3-3 for



عرض = w = 5m

$F_R = ?$ $x_p, y_p = ?$

$\rightarrow F_R = P_c \times A = \gamma h_c \times A = \gamma \times (3m) \times (4\times5)^{m^2} = 60\gamma$

$\rightarrow y_c =$ فاصله از محور x ها $= 6 = AB$

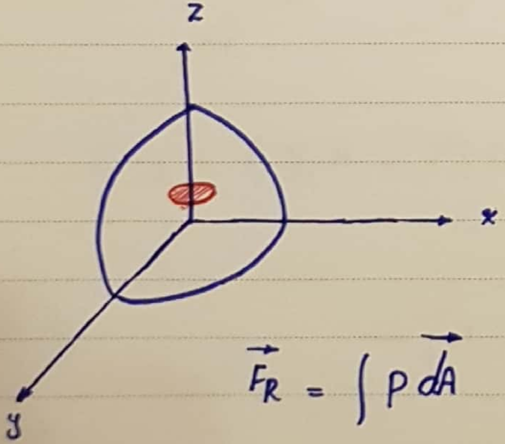
$\rightarrow y_p = y_c + \dfrac{I_G}{y_c A} = 6 + \dfrac{\frac{1}{12} b h^3}{6\times20} = 6 + \dfrac{\frac{1}{12}\times5\times4\times4\times4}{6\times20}$

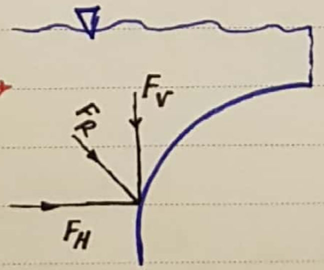
$= 6 + \dfrac{2}{9} = \dfrac{56}{9} = 6.22$

$\rightarrow x_c = x_p + \dfrac{\rho g \sin\theta \, I_{xy}}{F_R} \xrightarrow{I_{xy}=0} x_c = x_p$

$\rightarrow x_c = 2.5$

: Hydrostatic force on a Curved surface





$\vec{F_R} = \int p \, \vec{dA}$

F_H = Horizontal Force Component → is a force on a Projection surface

F_V = Vertical Force Component → is a weight of the liquid above the Curved Surface

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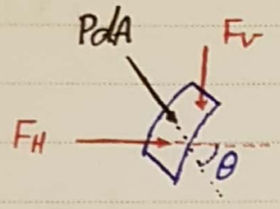
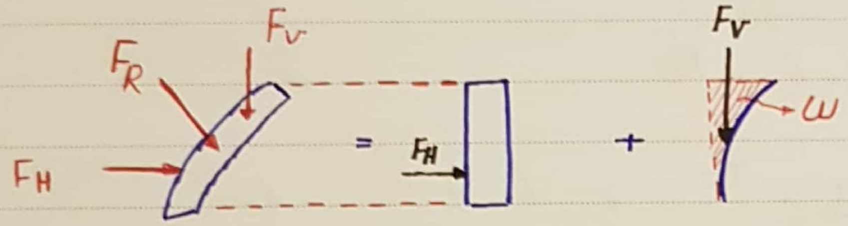


$$F_H = \int PdA\cos\theta = \int PdA_{vertical}$$

$$F_V = \int PdA\sin\theta = \int \gamma h \sin\theta\, dA = \gamma \int dV$$

$$= \gamma V$$



$$F_V \rightarrow x_p = \frac{1}{V}\int x\,dV$$

* اگر آب یا سیال درون سطح منحنی باشد نیرویی برابر با وقتی دارد که سیال در خارج از منحنی قرار دارد با این تفاوت که قرینه‌ی آن است.

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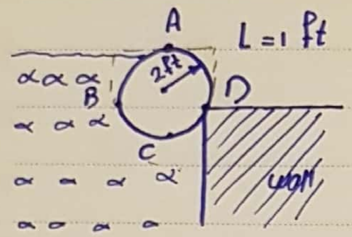
مثال)



$L = 1\ ft$

a) weight of the cylinder ?

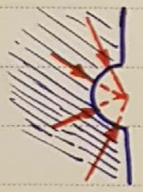
b) The force exerted against the wall ?

$$\uparrow F_{BCD} = \gamma \left(\frac{\pi r^2}{2}(L) + 2r(r)L \right) = (2\pi + 8)\gamma$$ (a

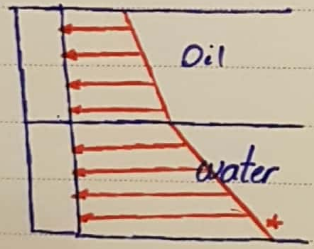
$$\downarrow F_{AB} = \gamma \left(r^2 - \frac{\pi r^2}{4} \right) = (4 - \pi)\gamma$$

$$\longrightarrow F_V = F_{BCD} - F_{AB} = 838\ lb$$

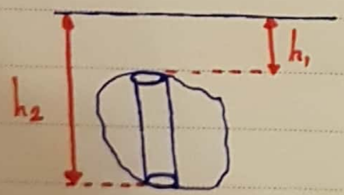
$$F_H = \gamma (h_c) A = \gamma \left(\frac{r}{2}\right) A = \gamma \left(\frac{r}{2}\right) \times r \times l = 2\gamma$$ (b



* محل برایند نیروها حول نیم دایره (مرکز کره) است.



* پروفایل فشار



$$dF = \rho g (h_2 - h_1)\, dA \longrightarrow F = \rho g \forall$$

$(h_2 - h_1)\,dA = d\forall$

F → buoyancy force

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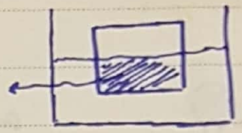
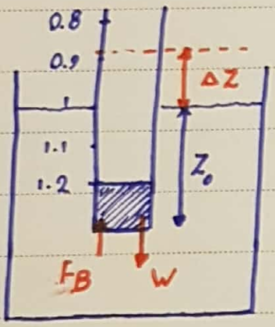
* $F_B = \rho_f g \forall_{sub} = W = \rho_{body}\, g \forall_{total}$



$$\Rightarrow \boxed{\frac{\rho_b}{\rho_f} = \frac{\forall_{sub}}{\forall_{total}}}$$

- $\rho_b < \rho_f$ — Submerge body
- $\rho_b = \rho_f$ — immerse body
- $\rho_b > \rho_f$ — Sinked body

Hydrometer:



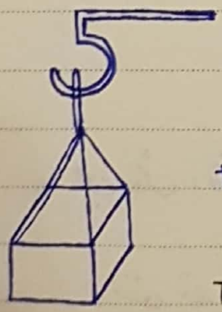
(1) $F_B = W_{Hydrometer} = \rho_w g \forall = \rho_w g (A Z_0)$

(2) $\rho_f < \rho_w \Rightarrow F_B = W_{Hydrometer} = \rho_f g (A(Z_0 + \Delta Z))$

$$\Rightarrow \rho_w g A Z_0 = \rho_f g A (Z_0 + \Delta Z) \Rightarrow \boxed{\frac{\rho_f}{\rho_w} = \frac{Z_0}{Z_0 + \Delta Z} = S.G}$$

* برای اندازه گیری چگالی جامدات از Pycnometer استفاده می‌شود

(3) $\rho_f > \rho_w \Rightarrow \boxed{S.G = \frac{Z_0}{Z_0 - \Delta Z}}$



مثال:

$\forall = 0.4 \times 0.4 \times 3$ $\qquad \rho_{block} = 2300 \frac{kg}{m^3}$ $\qquad \rho_{Seawater} = 1025 \frac{kg}{m^3}$

الف: T وقتی بلاک در هوا است؟ $T = mg = \rho \forall g = 2300 \times 0.4 \times 0.4 \times 3 \times 10 = 10.8\ kN$

ب: T وقتی در آب دریا است؟ $T = W - F_B = 10.8\,kN - (1025 \times 0.4 \times 0.4 \times 3 \times 10) = 6\ kN$

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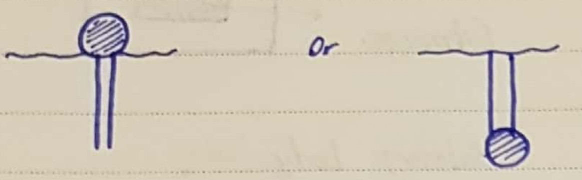
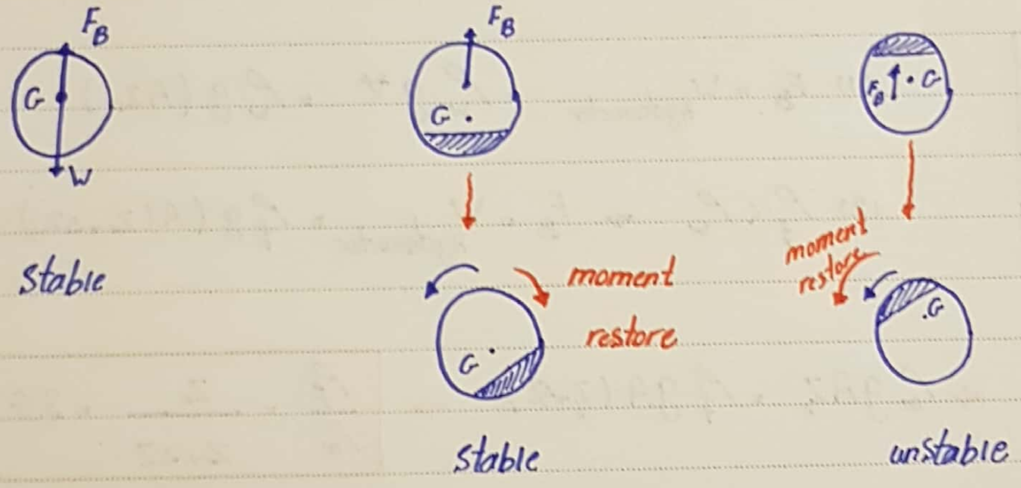
Stability :



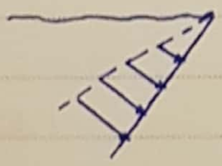
مرکز شناوری : $xF = \int x\, dF \rightarrow x(\gamma \forall) = \int x \gamma\, d\forall$

$$\Rightarrow \boxed{x_b = \frac{1}{\forall}\int x\, d\forall}$$



Pressure Prism :

γh همان h است



$dF = \gamma h\, dA = dV \longrightarrow F = V = \frac{1}{2}\gamma h b h$

* این روش ، روش منشور فشار است ، و فقط برای دریچه های تخت کاربرد دارد و نمی توان از منشور فشار برای دریچه های منحنی استفاده کرد .

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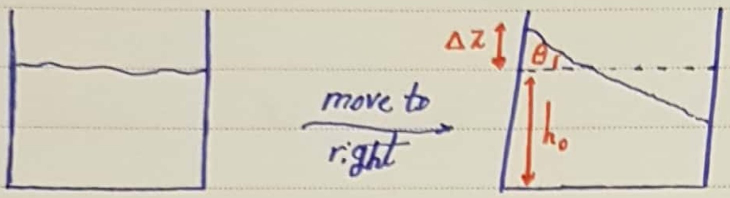
Rigid body motion in fluids:

$$-\nabla P + \rho g \hat{k} = \rho \vec{a} = \frac{dF}{d\forall}$$



$$\vec{a} = a_x \hat{i} + a_z \hat{k}$$

$$\frac{\partial P}{\partial x} = -\rho a_x$$

$$\frac{\partial P}{\partial y} = 0$$

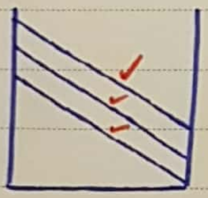
$$\frac{\partial P}{\partial z} = -\rho(g + a_z)$$

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz \longrightarrow dP = -\rho a_x dx - \rho(g + a_z)dz$$

$$\Rightarrow P = P_0 - \rho a x - \rho(g + a_z)Z$$

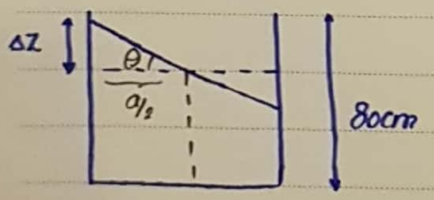
Surfaces of Constant Pressure:

$$dP = 0 \rightarrow -\rho a_x dx = \rho(g + a_z)dz \rightarrow \frac{dz}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$



✓: در این خطوط فشار ثابت است.

مثال:



2m×0.6m

0-90 km/h → 10s

$V = at + V_0 \rightarrow 25 = 10a \rightarrow a = 2.5\ m/s$

$$\frac{dz}{dx} = -\frac{a_x}{g + a_z^0} = -\tan\theta \rightarrow \theta = 14.31$$

$\Delta Z = \frac{b}{2}\tan\theta \rightarrow \Delta Z_1 = 25.5\ cm \leftarrow b = 2m$

$\Delta Z_2 = 7.6\ cm \leftarrow b = 0.6m$

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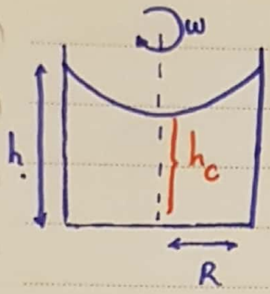


$$\begin{cases} P = P_0 + \dfrac{\rho \omega^2}{2} r^2 - \rho g z \\ z = \dfrac{\omega^2}{2g} r^2 + h_c \end{cases}$$

Chapter 3 : fluid flows

$\vec{V} = V(x, y, z)$

- در این فصل به جای سرعت از بردار سرعت استفاده می کنیم.

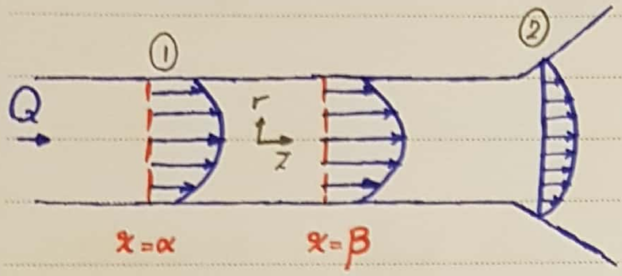
$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

steady flow → $\dfrac{\partial n}{\partial t} = 0$

uniform flow → $\dfrac{\partial n}{\partial x} = 0$

flow → viscid $\mu \neq 0$

flow → inviscid $\mu = 0$ ⟶ ideal flow



① ⟶ $u = u(r)$ 1D flow

② ⟶ $u = u(r, z)$ 2D flow

Steamline :

a Curve where is tangent to instantaneous velocity vector

$d = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$V = u\hat{i} + v\hat{j} + w\hat{k}$

برای موازی بودن ⟶ $\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Pass line :

The actual Path that traveled by fluid Particle.

(بدون بعد) Reynolds = $Re = \frac{\rho \times \vec{V} \times L}{\mu}$ (L: قطر در لوله)

Re → laminar $Re < 2100 - 2300$

Re → Turbulen $Re > 4000$

$\rho = cte$ ⟶ incompressible ⟶ liquid

$\rho \neq cte$ ⟶ Compressible ⟶ gas

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- system → flexible, identifiable quality of mass ✓
- Control volume → an arbitary (اختیاری) volume in space through which fluid flow.

✓ هیچ جرمی از مرزهای سیستم عبور نمی کند.

- Conservation of mass → $\left(\frac{dM}{dt}\right)_{sys.} = 0$ $M = \int dm = \int d\forall \rho$

- Newton's Second law → $P = \int \vec{V} dm$, $F = \frac{dP}{dt}$

$$P = \int \vec{V} \rho d\forall$$

- First law of thermodynamic :

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

$$E_{sys} = \int e\, dm = \int e \rho d\forall$$

(e → خاصیت شدتی)

- 2^{nd} law of thermodynamic :

$$S_{sys.} = \int s\, dm = \int s \rho d\forall$$

$N_{sys} = \int \eta \rho d\forall$

- $N = M \rightarrow \eta = 1$
- $N = P \rightarrow \eta = \vec{V}$
- $N = E \rightarrow \eta = e$
- $N = S \rightarrow \eta = s$

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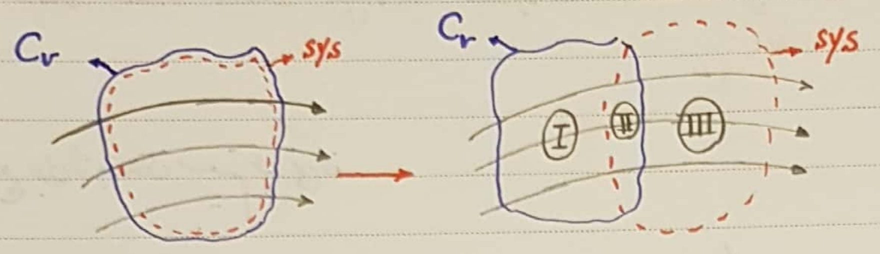
$$\star\left(\frac{dN}{dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{CV} \eta \rho\, d\forall + \int_{CS} \eta\, \rho \vec{V}\cdot d\vec{A}$$

└ Scaler Product



اثبات ←

$$\left(\frac{dN}{dt}\right)_{sys} = \lim_{\Delta t\to 0} \frac{N_s)_{t_0+\Delta t} - N_s)_{t_0}}{\Delta t}$$

$N_s)_{t_0} \to N_{cv})_{t_0}$

$$N_s)_{t_0+\Delta t} = (N_{II} + N_{III})_{t_0+\Delta t} = (N_{cv} - N_I + N_{III})_{t_0+\Delta t}$$

$$\to \dots \to \left(\frac{dN}{dt}\right)_{sys} = \lim_{\Delta t\to 0} \frac{(N_{cv})_{t_0+\Delta t} - (N_{cv})_{t_0}}{\Delta t} + \lim_{\Delta t\to 0} \frac{N_{III}}{\Delta t} - \lim_{\Delta t\to 0} \frac{N_I}{\Delta t}$$

$$\frac{\partial N_{cv}}{\partial t} = \frac{\partial}{\partial t}\int \eta \rho\, d\forall$$

$$\int_{III} \eta \rho \vec{V}\cdot d\vec{A} + \int_{I} \eta \rho \vec{V}\cdot d\vec{A} = \int_{CS} \eta\, \rho \vec{V}\cdot d\vec{A}$$

$$\Rightarrow \left(\frac{dN}{dt}\right)_{sys.} = \frac{\partial}{\partial t}\int_{CV} \eta \rho\, d\forall + \int_{CS} \eta\, \rho \vec{V}\cdot d\vec{A}$$

→ Conservation of mass → $\frac{\partial}{\partial t}\int \rho\, d\forall + \int \rho \vec{V}\cdot d\vec{A} = 0$

steady state → $\int \rho \vec{V}\cdot d\vec{A} = 0$

incompresible → $\int \vec{V}\cdot d\vec{A} = 0$

uniform جریان → $\sum \vec{V}\cdot A = 0$

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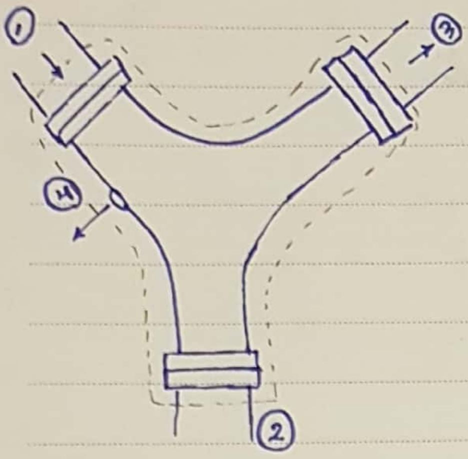
مثال ۱ Fox 4.1



1) st.st , incompressible , uniform

2) $A_1 = 0.2m^2$ $A_2 = 0.2m^2$ $A_3 = 0.15m^2$

3) $Q_4 = 0.1 \frac{m^3}{s} = V \times A$

4) $V_1 = 5 \, m/s$ $V_3 = 12 \, m/s$

$V_2 = ?$ ←

فرض کنیم جریان 2 خروجی باشد.

$\rightarrow -V_1A_1 + V_2A_2 + V_3A_3 + V_4A_4 = 0$

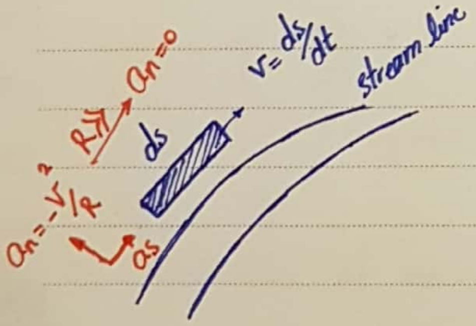
$\rightarrow V_2 = -4.5 \, m/s$

بنابراین جریان 2 ورودی است

$N = P \longrightarrow \eta = V$

$$\Rightarrow F = \frac{dP}{dt} = \frac{\partial}{\partial t}\int V\rho d\forall + \int V\rho \vec{V}\cdot\vec{dA} \longrightarrow P = \int V dm$$

معادله 4.18 :



$a_s = \frac{dV}{dt} \longrightarrow V = V(s,t)$

$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\cdot\frac{\partial s}{\partial t} + \frac{\partial V}{\partial t}\cdot\frac{\partial t}{\partial t}$

$= V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$

$\xrightarrow{st.st} a_s = V\frac{\partial V}{\partial s}$

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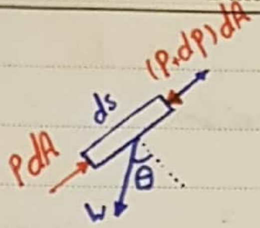

1) along with the Streamline

2) No friction

3) Steady flow

4) incompressible

→ $PdA - (P+dP)dA - W\sin\theta = ma = mv\frac{dv}{ds}$

→ ... → $\frac{dP}{\rho} + v\,dv + gz = 0$ → $\frac{dP}{\rho} + \frac{1}{2}dv^2 + g\,dz = 0$

→ $\int \frac{dP}{\rho} + \frac{v^2}{2} + gz = cte$

⇒ $$\boxed{\frac{P}{\rho} + \frac{v^2}{2} + gz = cte}$$

→ $\frac{P}{\rho g} + \frac{v^2}{2g} + z = cte = H$

pressure head ($\frac{P}{\rho g}$) — velocity head ($\frac{v^2}{2g}$)

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